

[This question paper contains 6 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 1564 A  
Unique Paper Code : 42351201  
Name of the Paper : Calculus and Geometry,  
CBCS (LOCF)  
Name of the Course : B.Sc. (Programme)  
Mathematical Sciences /  
Physical Sciences  
Semester : II  
Duration : 3 Hours Maximum Marks : 75

**Instructions for Candidates**

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. This question paper has six questions in all.
3. Attempt any two parts from each question.
4. All questions are compulsory.
5. Marks are indicated.

1. (a) Sketch the graph of the function  $f(x) = x^4 + 2x^3$ .  
(6.5)

P.T.O.

- (b) Sketch a graph of a function  $f$  with all the following properties :

The graph has  $y = 1$  and  $x = 3$  as asymptotes;  $f$  is increasing for  $x < 3$  and  $3 < x < 5$  and is decreasing elsewhere; the graph is concave up for  $x < 3$  and for  $x > 7$  and concave down for  $3 < x < 7$ ;  $f(0) = 4 = f(5)$  and  $f(7) = 2$ . (6.5)

- (c) Identify the symmetries of the curve  $r = 1 + 2 \cos \theta$  and then sketch the curve. (6.5)

- (d) Let  $x = 4 \tan 2t$ ,  $y = 3 \sec 2t$  where  $0 \leq t \leq \pi$ . Find an explicit relation between  $x$  and  $y$ . Also, sketch the path described by the given parametric equations over the prescribed interval. (6.5)

2. (a) Evaluate the following limits using L'Hospital's Rule

$$\lim_{x \rightarrow \pi/2} (\tan x)^{(\pi/2) - x} \text{ and } \lim_{x \rightarrow 0^+} [\tan x \log x]. \quad (6)$$

- (b) Sketch the graph of  $y = (x - 4)^{2/3}$ . (6)

- (c) Use cylindrical shells to find the volume of the solid that is generated when the region  $R$  in the first quadrant enclosed between  $y = x$  and  $y = x^2$  is revolved about the line  $x = 0$ . (6)

(d) Find the volume of the solid formed when the region between the graphs of  $y = 1 + 2x^2$  and  $y = 3 - 2x^2$  is revolved about the x-axis. (6)

3. (a) Find the volume of the solid generated when the region enclosed by  $x = 0$ ,  $y = 0$ ,  $x = 1$  and  $y = x^2 + 1$  is revolved about the y-axis. (6.5)

(b) Find the length of the curve  $y = 2x^2 + 1$  over the interval  $[1, 3]$ . (6.5)

(c) Find the area of the surface swept out by revolving  $y = \sqrt{9 - x^2}$  about the x-axis. (6.5)

(d) Find the arc length of the curve  $x = t^3$ ,  $y = 3\frac{t^2}{2}$ ,  $0 \leq t \leq \sqrt{3}$ . (6.5)

4. (a) Derive the reduction formula

$$\int \sin^n x \, dx = -\frac{1}{n} \sin^{n-1} x \cos x + \frac{n-1}{n} \int \sin^{n-2} x \, dx$$

and evaluate the integral  $\int_0^{\pi/2} \sin^5 x \, dx$ . (6)

(b) Prove that for nonnegative integers  $m$  and  $n$ ,

$$\int_0^{2\pi} \cos mx \cos nx \, dx = 0. \quad (6)$$

(c) Obtain reduction formula

$$\int \tan^n x \, dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x \, dx, \quad n > 2$$

and hence using that evaluate the integral

$$\int \tan^4 x \, dx. \quad (6)$$

(d) Evaluate  $\int_0^{\pi/2} \sin^2 \frac{x}{2} \cos^2 \frac{x}{2} \, dx$  after obtaining the reduction formula for the integral

$$\int \sin^m x \cos^n x \, dx. \quad (6)$$

5. (a) Describe the graph of the equation

$$y = 4x^2 + 8x + 5. \quad (6.5)$$

(b) Find equation for the hyperbola that has vertices  $(0, \pm 2)$  and asymptotes

$$y = \pm \frac{2}{3}x. \quad (6.5)$$

- (c) Sketch the graph of ellipse and label the foci, vertices and ends of the minor axis

$$(x + 3)^2 + 4(y - 5)^2 = 16. \quad (6.5)$$

- (d) Rotate the axes of coordinates to get rid of the  $xy$ -term from the equation, name the conic  $x^2 + 4xy - 2y^2 - 6 = 0$  and sketch its graph.

(6.5)

6. (a) Show that the graphs of given  $r_1(t)$  and  $r_2(t)$  intersect at the point  $P(1, 1, 3)$ . Find the acute angle between the tangent lines to the graphs of  $r_1(t)$  and  $r_2(t)$  at this point, where

$$r_1(t) = t^2 \hat{i} + t \hat{j} + t^3 \hat{k}$$

$$r_2(t) = (t-1) \hat{i} + \frac{1}{4} t^2 \hat{j} + (5-t) \hat{k}. \quad (6)$$

- (b) Sketch the graph and show direction of increasing  $t$  for

$$r(t) = 9 \cos t \hat{i} + 4 \sin t \hat{j} + t \hat{k}. \quad (6)$$

- (c) Evaluate  $\nabla \times (\nabla U \times \nabla V)$  where  $U = x^2 yz$ ,  
 $V = xy - 3z^2$ . (6)

(d) Show that divergence of the field

$$F(x, y, z) = \frac{c}{(x^2 + y^2 + z^2)^{3/2}} (x\hat{i} + y\hat{j} + z\hat{k}) \text{ is zero.}$$

(6)

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